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DISTRIBUTIONAL ERRORS IN RELIABILITY

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Abstract—The complexity of distributional error analyses has handicapped the revival of analytical methods of reliability being based on the exponentiality of time to unit failure and, most often, repair. The assertation, that these hypothesis hardly affect the calculation of reliability of strongly redundant systems with reliable units, is substantiated by studying parallel systems with identical units. As a by-product a surprising bridge between reliability and steady-state availability is established. This result is obtained both by Markovian theory and methods free of any distributional hypothesis.

NOTATION

A steady-state availability
D $(1 + T_1/T_2)^n$
n number of units in the system
p_k steady-state Markovian probability of $k = 0, \ldots, n$ failed units
$R(t)$ reliability, probability of no system failure up to time t
T_0 steady-state mean time between system failures
$T_{1,2}$ mean time to unit failure, repair.

RESULTS

Consider parallel structures with identical units. Gnedenko [1, pp. 333–334] obtained, without making any distributional hypothesis, the stationary mean time between system failures and reliability as

$$T_0 = T_2(D - 1)/n, \quad R_1(t) \cong \exp(-t/T_0). \qquad (1)$$

If $D \gg 1$, $R_1(t)$ is accurate and the difference between T_0 and mean time before first system failure is negligible. Reliability and steady-state availability are bridged by noting D as the inverse of stationary unavailability

$$A = 1 - (1 - T_1/(T_1 + T_2))^n, \quad D = (1 - A)^{-1}. \qquad (2)$$

This is no coincidence. Under Markovian assumptions of exponential time to unit failure and repair the probability that the system fails from a state of less than $n - 1$ failed units during short mission time t can be discarded. Thus,

$$R_3(t) \cong 1 - p_{n-1}t/T_1 = 1 - nt(T_2)^{n-1}/(T_1 + T_2)^n = 1 - nt/(DT_2) \qquad (3)$$

This formula should not be applied when $t \geqq 0.3DT_2/n$.

When t is small compared with T_0, equation (1) almost coincides with equation (3):

$$R_1(t) \cong 1 - t/T_0 = 1 - nt/((D - 1)T_2). \qquad (1')$$

The relative error of unreliability between equations (3) and (1), defined as $G(t) = |R_3(t) - R_1(t)|/(1 - R_3(t))$, is increasing in time but decreasing in both D and n for a broad range of values. None of these properties always holds. $R_1(t)$ and $R_3(t)$ intersect in the neighborhood of T_2.

Both formulas tend to be conservative, the former, because T_0 is less than mean time to first system failure, the latter for reasons already explained.

Example

A marine radar has $T_1 = 800$ hr and $T_2 = 10$ hr. For $n = 2, 3, 4$, $D = 6561$, $531{,}441$, $43{,}046{,}721 \gg 1$; $T_0 = 3.74, 202, 12{,}277$ yr. $G(1\text{ yr}) = 0.12, 0.002, 0.00004$.

Any series–parallel system can be evaluated in this way by considering the series as single units. If the units are not identical, an upper bound for the distributional error is obtained by choosing the weakest unit in place of the other units. The calculation of reliability of any n-configuration that has a greater redundancy than the corresponding n-parallel system is less affected by distributional hypothesis. Our method can be made to cover almost any system encountered in practice.

REFERENCE

B. V. Gnedenko, Yu. K. Belyayev and A. D. Solovyev, *Mathematical Methods of Reliability*, Academic Press, New York (1969).