Reliability Models for Radar Systems

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Abstract - Reliability is treated as a time dependent function of performance. We separate the probability $p_k(t)$ of k units under simultaneous renewal at time t, and the conditional Cdf, $S_k(z,t)$, of the loss of performance z. An attempt is made to justify the use of Markov models in calculating $p_k(t)$ even when n, the size of system, is only two.

Key words - reliability, radar

Reader Aids:

Purpose: Present easy-to-use results

Special math needed for explanations: Probability

Special math needed for results: Same

Results useful to: Reliability engineers

1. Introduction

The fundamental concept of this article is performance. It might be defined as the space in which objects with certain properties can be detected under specified circumstances with a threshold confidence. A radar is said to have failed after a critical loss of performance. This is preceded by a process of, say, accumulation of stepwise injuries and continous wear-and-tear.

It is assumed that the maintainer is less superficial than the author; he is supposed to know utility criteria for performance, failure and perhaps the Cdf or its first two moments. System performance is naturally defined as the superposition of all unit measures. In the most trivial case it could be considered as a sum of binary-coded matrices. This can, however, serve only as lower bound for the superposition of several null elements may yield a unit element.

The Cdf of the loss os system performance is

(1) $$F(z,t) = \sum_{k=0}^{n} p_k(t)\ S_k(z,t).$$

The corresponding steady state Cdf, if it exists, is

(2) $$F(z) = \sum_{k=0}^{n} p_k\ S_k(z).$$

Let V denote maximum system capacity, and (1-c)V with $0 \leq c < 1$ stand for the minimum required performance. Then system point availability and uptime ratio are

(3) $$A(t) = F(cV,t) \text{ and } UTR = F(cV).$$

By using the expression 'up to time t' instead of 'at time t' we obtain the more complicated measure of network reliability, From now on let us concentrate upon the terms $p_k(t)$.

The vast majority of radar systems consist of only a few units, so that there is room for ambitious models based on exact distributional behaviour. Besides, most reliability configurations reduce to combinations of sets in parallel or series. There are, however, some quite large networks such as DEW and Pinetree lines with 31 and 25 sets, respectively, not to speak of PVO-Strany.

Data provided to the author by the Electrotech. Dep of the Finnish H.Q. seem to indicate that failure rates are nonincreasing and nearly Weibull distributed, while downtimes involve many complications that can be solved by using mixtures of two or even three two-parameter Weibull Cdf, the parameters of which are estimated by minimizing chisquare. We proceed to show that whatever is the nature of these Cdf, we may base our models on exponentiality. Parallel systems have been chosen for the sake of simplicity and to avoid making any distributional hypothesis.

2. Parallel Systems

Let T_1 and T_2 denote unit MTBF and MTTR. By [1,333-4] the mean time between system failures is

(4) $$T_o = T_2((1+T_1/T_2)^n-1)/n.$$ Moreover

(5) $$R(t) \cong \exp(-t/T_o).$$

If $D = (1+T_1/T_2)^n \gg 1$, this approximation is close and the difference between T_o and system MTBF neglible.

We established with the kind aid of Machinery Co, the local importer, that Raytheon/SELENIA 1660TM has an average T_1=780h and (i) T_2=9.9h in port and (ii) T_2=115h at sea. These values are used in our examples. It is likely that D is going to increase. Even such sophisticated units as ASR-3 (Westinghouse Defence) have T_1=750h or more. Let us compare (5) with the corresponding Markov result (M) [2,193-5] when n=2. Furthermore, for small values of t, the following steady state approximation is applicable

(6) $$R(t) \cong 1 - p_{n-1}t/T_1 = 1 - ntT_2^{n-1}/(T_1+T_2)^n$$

Thus for n=2 and t=20,200,1000,3000,8766h(1y) we have

	(5):	D=6366, T_o=31510h,	R(t)=.9994,.9937,.9688,.9092,.7571
(i)	(M):	MTBF = 31900h,	R(t)=.9996,.9940,.9694,.9105,.7599
	(6):		R(t)=.9994,.9937,.9683,.9048,.7218
	(5):	D=60.6, T_o=3425h,	R(t)=. 994,. 943,. 747,. 417,. 077
(ii)	(M):	MTBF = 3815h,	R(t)=. 999,. 967,. 782,. 458,. 098
	(6):		R(t)=. 994,. 943,. 713, ---------.

It is no surprise that (5) is more conservative than (M), as T_o is slightly less than (true) MTBF. In a certain sense (5) is most fundamental:
Suppose all units can be simultaneously repaired.
Then

(7) $$UTR = 1 - (1 - T_1/(T_1+T_2))^n \text{ and } D = (1-UTR)^{-1}.$$

It is perhaps of interest to note how fast A(t) approaches UTR in the Markovian case [2,(8.69)] when n=2 and, say, both units have repair facilities.

Let $H = (A(t)-UTR)/(1-UTR)$. Then $H < .25, 3.10^{-9}$ for $t > 20, 200$h and $H < .26, 10^{-4}$ for $t > 200, 1000$h in cases (i) and (ii) respectively.

3. General Networks

Assume that there are k+1 different states of radar energization with failure intensities q^j, $j=0,1,\ldots,k$. Suppose that n_j represents the optimal number of units in state j, $\sum_{j=o}^{k} n_j = n$, and the higher value of j the greater priority of keeping the quota. Furthermore, at least m radars might be needed to operate in the full mode k, while r represents the number of repair facilities. To be on the conservative side, we allow fewer than m radars run; the system is not necessarily obsolete, although network requirements are not satisfied. Let w denote average repair intensity per facility. Before fixing w, the time to detect failure and get radar into waiting line or under repair should be taken into account as a suitable weighed average of different states of energization. Then our (m,n,k,r)-network has the following set of intensities

$$q_i = \begin{cases} \sum_{j=1}^{k} n_j q^j + (n_o - i) q^0 & \text{if } i \leq n_o \\ \sum_{j=e+1}^{k} n_j q^j + \left(\sum_{j=0}^{e} n_j - i \right) q^e & \text{if } \sum_{j=0}^{e-1} n_j < i \leq \sum_{j=0}^{e} n_j \end{cases}$$

(8) where $e=1,2,\ldots,k$ and

$$w_i = \begin{cases} iw & \text{if } i < r \\ rw & \text{otherwise} . \end{cases}$$

By applying (8), a multitude of system measures are obtained in a similar fashion as in [2,293].

Thus far it has been tacitly assumed that all units are identical, possibly allowing small variations of individual intensities. This can usually be achieved by treating different radars of the same site as single or by a division into subsystems. Barlow and Proschan [3] have considered another approach of wide utility. Suppose components have an exponential time-to-failure and arbitary time-to-repair distribution. Lek k be the number of minimal cut sets, n_j of components in the j'th minimal cut set with q_{ji} and w_{ji} referring to the failure and repair rates of the i'th component in the j'th minimal cut set. Then

(9) $$1-R(t) < t \sum_{j=1}^{k} \prod_{i=1}^{n_j} q_{ji} \, w_{ji} \left(\sum_{s=1}^{n_j} 1/w_{js} \right) .$$

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